Assignment Number 2

Stochastic Processes course,

Semester 1, 90-91

Science and Research Branch in Azad University

1. Random variables X and Y have the joint PMF

$$p_{X,Y}(x,y) = \begin{cases} c(x^2+y^2), & \text{if } x \in \{1,2,4\} \text{ and } y \in \{1,3\}, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?

(b) What is $\mathbf{P}(Y < X)$?

(c) What is $\mathbf{P}(Y > X)$?

(d) What is $\mathbf{P}(Y = X)$?

(e) What is $\mathbf{P}(Y = 3)$?

(f) Find the marginal PMFs $p_X(x)$ and $p_Y(y)$.

(g) Find the expectations $\mathbf{E}[X]$, $\mathbf{E}[Y]$ and $\mathbf{E}[XY]$.

(h) Find the variances $\text{var}(X)$, $\text{var}(Y)$ and $\text{var}(X+Y)$.

(i) Let A denote the event $X \geq Y$. Find $\mathbf{E}[X \mid A]$ and $\text{var}(X \mid A)$.

2.

A simple example of a random variable is the *indicator* of an event A, which is denoted by I_A:

$$I_A(\omega) = \begin{cases} 1, & \text{if } \omega \in A \\ 0, & \text{otherwise.} \end{cases}$$

(a) Prove that two events A and B are independent if and only if the associated indicator random variables, I_A and I_B are independent.

(b) Show that if $X = I_A$, then $\mathbf{E}[X] = \mathbf{P}(A)$.

3.

Random variables X and Y are distributed according to the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} ax, & \text{if } 1 \leq x \leq y \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

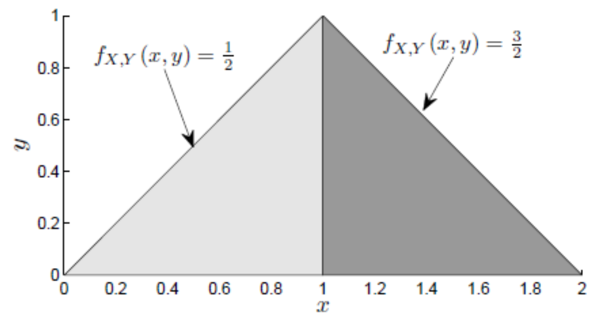
(a) Evaluate the constant a.

(b) Determine the marginal PDF $f_Y(y)$.

(c) Determine the expected value of $\frac{1}{X}$, given that $Y = \frac{3}{2}$.

4.

X and Y are continuous random variables. X takes on values between 0 and 2 while Y takes on values between 0 and 1. Their joint pdf is indicated below.



(a) Are X and Y independent? Present a convincing argument for your answer.

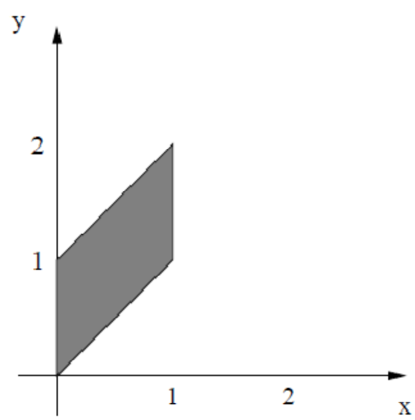
(b) Prepare neat, fully labelled plots for $f_X(x)$, $f_{Y|X}(y \mid 0.5)$, and $f_{X|Y}(x \mid 0.5)$.

(c) Let $R = XY$ and let A be the event $X < 0.5$. Evaluate $\mathbf{E}[R \mid A]$.

(d) Let $W = Y - X$ and determine the cumulative distribution function (CDF) of W.

5.

The random variables X and Y are described by a joint PDF which is constant within the unit area quadrilateral with vertices $(0,0)$, $(0,1)$, $(1,2)$, and $(1,1)$.



(a) Are X and Y independent?

(b) Find the marginal PDFs of X and Y.

(c) Find the expected value of $X + Y$.

(d) Find the variance of $X + Y$.